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COevolution and Self-organization
In dynamical Networks

Characterization of optimization as a driving force in network shaping

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Abstract

In D11 we dealt with networks that self-organized according to some microscopic rules. The macroscopic state is an emergent property of the non-lineal interaction between the units that form the system; in our case the network is characterized by some macroscopic observables: degree distributions, different sorts of measures of cycles, community distribution, and so on. Nevertheless, for some cases it is possible to design the proper structure of the network with some purpose, being this purpose local at the node scale or global at the network scale.

Along the project we have considered some dynamical properties of networks and have found the structures for which these properties are optimized. One of them is a problem of information flow and the other is related to synchronization. On the other hand, we have also tackled the issue of constructing networks according to some topological characteristics.

CONTENTS

INTRODUCTION

Along the current project we have followed different lines that are related to the generation of structures with prescribed characteristics that can play a crucial role in the dynamical properties of the network:

1.- a model of information flow in which search and congestion can be handled simultaneously

2.- generalizations of the configuration model: one in which clustering can be tuned in addition the degree distribution, another one in which uncorrelated scale-free networks are constructed, and finally, a third one, in which a kind of weighted configuration model is built.

3.- an analysis of synchronization in networks.

These lines are described in detail in the following sections.

INFORMATION FLOW: SEARCH AND CONGESTION

The problem of *searchability* in decentralized complex networks is of great importance in computer science, economy, and sociology. We have studied a formalism that is able to cope simultaneously with the problem of search and the congestion effects that arise when parallel searches are performed, and we obtain expressions for the average *search cost* both in the presence and the absence of congestion. This formalism is used to obtain optimal network structures for a system using a local search algorithm. It is found that only two classes of networks can be optimal: star-like configurations, when the number of parallel searches is small, and homogeneous-isotropic configurations, when it is large. [1].

It is clear that for the first case, when the load of packets in the network is low, it is a search problem; nevertheless, when the load is increased the problem becomes a congestion problem. This is the reason for which the two different structures appear in two different extreme configurations. In principle, this problem is a dynamical one and the search of optimal structures involves a complete dynamical simulation of the problem. One of our main findings has been the relation between the topological properties and the dynamical properties of the networks; this fact enables to optimize only topological features and hence no dynamical simulation is needed.

Once this formalism has been established we have considered different routes to generalize the results. On the one hand one can consider additional sources of information to handle the packets by the nodes. In this case the optimal structures are changed with respect to the previously discussed case[2]. On the other hand, the role played by the communities can also be considered as a new source of information. If the communities are sharply defined the nodes when processing the packets can use this topological characteristic efficiently.

NETWORK GENERATION

Most modelling efforts beyond the degree distribution have focused in the reproduction of two point correlations patterns, typified by the average nearest neighbour's degree, so that clustering is just obtained as a by-product. Those models do not allow much freedom in the form of the resulting clustering coefficient, neither in the ensuing degree distribution, so that, although a valuable first approach, they constitute a timid attempt as clustering generators. We have introduced a generator of random networks where both the degree-dependent clustering coefficient and the degree distribution are tuneable. Following the same philosophy as in the configuration model [3], the degree distribution and the clustering coefficient for each class of nodes of degree k are fixed *ad hoc* and *a priori*. The algorithm generates corresponding topologies by applying first a closure of triangles and secondly the classical closure of remaining free stubs. The procedure unveils a universal relation among clustering and degree-degree correlations for all networks, where the level of assortativity establishes an upper limit to the level of clustering. Maximum assortativity ensures no restriction on the decay of the clustering coefficient whereas disassortativity sets a stronger constraint on its behavior.

Correlation measures in real networks are seen to observe this structural bound. Overall, we hope that a more accurate shaping of synthetic networks will improve our understanding of real ones. At this respect, we believe our algorithm will be useful for the community working on complex networks science [4].

Another of the contributions of the consortium in the generation of prescribed complex networks concerns its degree correlations [5]. We have proposed a model capable of generating random uncorrelated scale-free networks with no multiple and self-connections. The model is based again in the configuration model [3] with an additional restriction on the maximum possible degree of the nodes.

Finally, an extension of the configuration model to weighted networks has also been worked out. In this case non-trivial correlations between strength and degree are generated [6].

OPTIMIZING SYNCHRONIZATION

A new family of graphs, entangled networks, with optimal properties in many respects, have been introduced. By definition, their topology is such that optimizes synchronizability for generic dynamical processes. These networks are shown to have an extremely homogeneous structure: degree, node-distance, betweenness, and loop distributions are all very narrow. They are characterized as well by a very interwoven (entangled) structure with short average distances, large loops, and no well-defined community-structure (poor modularity). More importantly, we show that this family of nets exhibits an excellent performance with respect to other connectivity or flow properties such as robustness against errors and attacks, minimal first-passage time of random walks, good searchability, efficient communication, etc. These remarkable features convert entangled networks in a powerful and useful concept, optimal or almost-optimal in many senses, and with plenty of potential applications in network design, computer science, or neuroscience [7].

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