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COevolution and Self-organization In dynamical Networks

Self-organized criticality in network formation

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Abstract

Complex network found in nature and society show lack of characteristic scales and have grown following rules that depend on the behavior of single nodes, not on the whole structure of the network. This is what makes many complex networks to be viewed as self-organized critical systems. These aspects are made evident usually in the form of power-law distributions of node connectivities; however, some other properties, as the size distribution of community sizes, have become quite important. On the other hand, simple self-organizing network models that can explain the characteristics of observed data are still necessary. This is in fact the goal of two of the works conducted by the consortium: one is a model of the evolution of the Internet at the Autonomous System level based on competition and adaptation; and the second one is a model of signalling networks for which the complex pattern of connexions is an essential ingredient for the emergence of complex dynamics. Finally, we have also focused on the self-organization of node properties keeping fixed the topology of the network, related to well-known models of neural networks.

CONTENTS

INTRODUCTION

The relation between self-organization and criticality received a great interest in the past decade since the pioneering work of Bak, Tang, and Wiesenfeld [1] that introduced the concept of self-organized criticality. Extremely simple models following its own rules are able to develop, in a self-organized way, into a complex statistical behaviour where there are no characteristic time or length scales, resembling those of well known physical properties of critical phenomena. For recent books on that subject see [2-4].

The models described in the previous paragraph were models embedded in fixed connectivity patterns. However, in the last years it has been shown that i) complex networks are everywhere around us; ii) they have grown in a self-organized way; iii) they show criticality in the sense of wide lack of characteristic scales.

Along the current project we have followed different lines that are related to the concept of self-organized criticality:
1.- Introducing new characteristics, although this line is related to Deliverable D04, of the networks that are scale-free. In particular, many social networks show distributions of community sizes that are power-law
2.- Introducing new models of network growing, that based only on the information available to the nodes, are able to develop complex patterns as observed in particular cases of social or biological networks.
3.- Introducing a model that describes accurately the evolution of the Internet by means of simple ideas of competition and adaptation.
4.- Showing that a model of neural networks, where nodes self-organize their synapses, is more efficient when the connectivity distribution is scale-free.

These lines are described in detail in the following sections.

HALLMARK OF SELF-ORGANIZATION IN COMPLEX NETWORKS

One of the goals of the new science of networks corresponds to the correct statistical characterization. Before the study of complex networks formed by thousands of nodes and links, the standard characterization of networks came from the field of social networks, where the main concern were to establish the role played by certain nodes in the global performance of the whole network. Nevertheless, when dealing with very large networks, the focus is more on the statistical properties of the components than on special roles. Thus, since the pioneering works of Watts and Strogatz [5] and Barabasi and Albert [6], the attention turned to compute statistical magnitudes such as the average length between nodes, average of different individual properties, distributions of properties as connectivities, and so on.

Along the present project, there have been several efforts in characterizing complex networks from a mathematical point of view, which has been the subject of deliverable D04. Nevertheless, there is point in this characterization that connects in a natural way with self-organization in critical systems. In most of the reported networks (natural, social, or technological) the degree distribution follows a power-law, which is a clear indication of criticality; and since, these networks have grown by means of simple rules, they are also self-organized.

The contribution of the project, as was reported in D04 is the analysis of the size distribution of community sizes [7,8,9]. Some networks that showing non-power law distributions of connectivities show, nevertheless, power law distributions of community sizes and topological self-similarity. Because of the analogy of this observation with this in river networks, where there exists a principle of optimization [10], it suggests that there could exist and underlying mechanism in the formation and evolution of social and natural networks.

MODELS OF NETWORK GROWING

Again, from the field of statistical physics one of the main contributions to complex networks has been the construction of very simple models that, although their simplicity, are able to reproduce the main features of complex networks. Thus, we could say that the first attempts where again those from Watts and Strogatz (small-world model) [5] and Barabasi and Albert (preferential attachment) [6].

Along this line the contribution of our project has been to introduce models that explain some features of complex networks, in particular of some social networks.

In [11] it was introduced a model in which hidden variables that were used to tag the vertices can determine completely the topological properties of the network, in particular degree correlations. Later on, we have focused on the specific features of social networks as compared to other networks appearing in nature or in technological fields: a large clustering coefficient, positive degree correlations, and community structure. By constructing a model based on the concept of social distance, which is in turn related to the location of the nodes in some "social space", we obtain some analytical results than are in very good agreement with a particularly large non-bipartite network, the Pretty-Good-Privacy web of trust [12].

Another contribution has been the construction of a self-organized model of collaboration networks. Again, a simple model with very simple rules is able to reproduce the specific features of this type of networks, in particular, the power law degree distribution. We can also find some analytical results, which compare with computer simulations and with available empirical data on real collaboration networks: movie-actor collaboration network, scientific collaboration networks, and board of directorships [13].

INTERNET EVOLUTION MODEL

The general approximation of networks as isolated systems, although possibly appropriate in some cases, must be overcame if we want to describe in a proper way complex systems which not generate spontaneously but self-organize within a medium in order to perform a function. Many networks evolve in an environment to which they interact and which usually provide the clues to understand functionality. Therefore, rules defined on the basis of internal mechanisms, such as preferential attachment that act internally at the local scale to connect nodes through edges, are not enough. When analyzing the dynamics of network assembly, the interlock of its constituents with the environment cannot be systematically obviated.

With the aim of approaching applicability and taking into consideration the arguments exposed above, we have proposed a new growing weighted network model[14]. The dynamical evolution is driven by growth, competition for resources and adaptation to the environment in order to maintain functionality in a demand and supply equilibrium, key mechanisms which may be relevant in a wide range of self-organizing systems, in particular those where functionality is tied to communication or traffic. The medium in which the network grows and to with it interacts can be represented by a pool of elements which, at the same time, provide resources to the constituents of the network and demand functionality, say for instance users in the case of the Internet or passengers in the case of any transportation network. Competition is here understood as a struggle between network nodes for new resources and is modelled as a rich get richer or preferential attachment process. For their part, this captured elements demand functionality so that nodes must adapt in order to perform efficiently. This adaptation translates into the creation of weighted links between nodes.

This externally driven growing weighted network model provides deeper insights on the understanding of the dynamical processes driving the network assembly of self-organizing complex systems. In particular, for the Internet at the Autonomous System level, it nicely reproduces most of the observed topological features offering at the same time an explanation to them.

EMERGENCE OF COMPLEXITY IN SIGNALLING NETWORKS

A variety of physical, social and biological systems generate complex fluctuations with correlations across multiple time scales. Intriguingly, in physiologic systems these long-range correlations are altered with disease and aging [15,16]. Such correlated fluctuations in living systems have been attributed to the interaction of multiple control systems, however, the mechanisms underlying this behaviour remain unknown .

We have shown that correlated fluctuations characterized by 1/f scaling of their power spectra can emerge from networks of simple signalling units [17]. We find that the generation of such long-range correlated time series requires: i) a complex topology with a discrete and sparse number of random links between units, ii) a restricted set of nonlinear interaction rules, and iii) the presence of noise. Moreover, we find that changes in one or more of these properties leads to degradation of the correlation properties. These findings may help in elucidating the genesis of complex signals in physiology and their alterations with age and disease [15,16].

SELF-ORGANIZATION OF NODE PROPERTIES

Another line of research that fits into the present deliverable is the study of the performance of neural automata with complex patterns of connectivity. In particular, in [18] it is shown that the capacity to store and retrieve binary patterns is higher for attractor neural networks with scale-free topology than for highly random-diluted Hopfield networks with the same number of synapses. Moreover, the performance of scale-free networks increase with the exponent of the power-law distribution of connectivities, which show the important role that in this physical mechanism is played by the hubs (the nodes with the highest number of links) of the network.

In another contribution [19], the effect of dynamic synapses in scale free network topologies is considered. Again, a power-law distribution of connectivities makes the system to outperform diluted networks, in the sense that networks are more robust and efficient in the retrieval of information.

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