

## Efficiency and reliability of epidemic data dissemination in complex networks

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We study the dynamics of epidemic spreading processes aimed at spontaneous dissemination of information updates in populations with complex connectivity patterns. The influence of the topological structure of the network in these processes is studied by analyzing the behavior of several global parameters, such as reliability, efficiency, and load. Large-scale numerical simulations of update-spreading processes show that while networks with homogeneous connectivity patterns permit a higher reliability, scale-free topologies allow for a better efficiency.

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Modern society increasingly relies on large-scale computer and communication networks, such as the Internet. A major challenge in these networks is the development of reliable algorithms for the dissemination of information from a given source to thousands, or even millions, of users, such as for news and stock exchange updates, mass file transfers, and Internet broadcasts [1,2]. In epidemic-inspired communication, this is achieved by exploring a mechanism analogous to the spreading of infectious diseases in populations [3,4]. The information spreads like a benign epidemic through local interaction between nodes which forward the message they receive to a random selection of their peers in the network, until the whole system becomes “infected” with information. The great advantages of epidemic-style communication is that dissemination proceeds on a local basis, without any coordination from a central organizing body [3,4]. These protocols are also highly resilient to sudden failure of communication links and nodes.

A relevant result in the mathematical theory of epidemics is that the spreading of infection in a population is strongly affected by the patterns of connectivity in the underlying contact networks. In particular, in scale-free topologies, characterized by degree distributions with power-law behavior  $P(k) \sim k^{-\gamma}$  [5], the statistical relevance of hubs makes the network highly permeable to attacks [6–8] and the spreading of infections [9] and highlights the need for special immunization strategies. This result suggests that the topology of the underlying computer and communication network might heavily affect the performance of epidemic-style data dissemination protocols. Surprisingly, however, the impact of network topology on such protocols has not been thoroughly explored, although the results could have an important technological value. Indeed, these protocols can potentially find a large spectrum of application, such as mobile communication networks and, more recently, resource discovery in the so-called peer-to-peer systems built on top of the Internet [10], and finally in grid computing [11].

In this paper, we define a simple epidemic data dissemination model and perform a detailed numerical study of the dynamics of the information propagation in networks with

diverse topological properties. Our basic model is a slightly modified version of the Daley and Kendall (DK) model [12–14] and it can be considered as the simplest epidemic algorithm for the updating of distributed databases [4,15]. We study the main relevant features of the model, such as the reliability of the dissemination process and the amount of traffic generated by the dynamics. Specifically, we study two different prototypical networks: a random homogeneous network [16] and a scale-free network. The results obtained point out that in the homogeneous topology, the epidemic process provides a more reliable updating of the network. The scale-free topology, on the other hand, allows the algorithm to perform more efficiently in terms of the generated traffic. Finally, we compare the present model with a deterministic broadcast process and find that in a wide range of the model parameters the epidemic algorithm is more efficient.

The model we shall consider is defined in the following way. Each of the  $N$  elements of the network can be in three possible states. We call a node holding an update and willing to transmit it a *spreader*. Nodes that are unaware of the update will be called *ignorants* while those that already know it but are not willing to spread the update anymore are called *stiflers*. We denote the density of ignorants, spreaders, and stiflers at time  $t$  as  $\psi(t)$ ,  $\phi(t)$ , and  $s(t)$ , respectively, such that for all  $t$ ,  $\psi(t) + \phi(t) + s(t) = 1$ . The spreading process takes place along the links between spreaders and ignorants. Each time step, spreaders contact one (or more) neighboring node. When the spreader contacts an ignorant, the last one turns into a new spreader at a rate  $\lambda$ . On the other hand, the spreader becomes a stifler with rate  $1/\alpha$  if a contact with another spreader or a stifler takes place [17]. The parameter  $\alpha$  can be considered as the average number of contacts with spreader/stifler nodes before the spreader turns itself into a stifler. This dynamics mimics the attempt to diffuse an update or rumor by nodes which have been recently updated. At the same time, if a node attempts too many times to communicate the update to nodes which have already received it, it stops the process, turning itself into a stifler. In other words, the node realizes that the update has lost its novelty and

TABLE I. Reliability of the epidemic process, defined as the density of nodes that has received the update, in the WS and BA networks for different values of the parameter  $\alpha$ . Standard deviation of the mean values is  $\pm 2$  units in the last significant digit.

$\alpha$	1	2	3	4	5	6	8	9
$R_{WS}$	0.831	0.962	0.986	0.994	0.996	0.996	0.998	0.999
$R_{BA}$	0.368	0.684	0.781	0.874	0.932	0.952	0.977	0.987

becomes uninterested in diffusing it. The present dynamics thus introduces a tradeoff in maximizing the number of updated nodes and minimizing the number of contacts attempted. Obviously, the efficiency of the spreading process will depend on the rate at which individuals lose interest in further spreading of the rumor and the topology of the underlying network.

At the mean-field level and using the homogeneous assumption for the network connectivity pattern, the time evolution of ignorants, spreaders, and stiflers is described by the simple set of equations

$$\partial_t \psi(t) = -\lambda \psi(t) \phi(t), \quad (1)$$

$$\partial_t \phi(t) = +\lambda \psi(t) \phi(t) - \frac{1}{\alpha} \phi(t) [\phi(t) + s(t)], \quad (2)$$

where  $s(t)$  is obtained by the normalization condition  $s(t) = 1 - \psi(t) - \phi(t)$  [12,14]. The dynamics asymptotically evolves to the state  $\phi(\infty) = 0$  in which the system is frozen. Noticeably, in random homogeneous networks, the density  $s(\infty)$  of elements which are aware of the update is always a finite fraction of the whole population [12,14]. The homogeneous assumption is, however, not valid anymore in the case of heterogeneous scale-free networks, where it is known that spreading processes may show very different properties [9]. In particular, an explicit dependence on the nodes' degree  $k$  must be included in the rate equations. While a general analytical solution cannot be obtained in this case, numerical studies on scale-free networks can be used to evaluate the reliability and efficiency of this process in more complex topologies [12,13].

In the present investigation, we used two specific network models. First we consider the Barabási-Albert (BA) network [5]. In this model, starting from a set of  $m_0$  nodes, one preferentially attaches at each time step a newly introduced node to  $m$  older nodes. The procedure is repeated many times and a network with a power-law degree distribution  $P(k) \sim k^{-\gamma}$  with  $\gamma=3$  and average connectivity  $\langle k \rangle = 2m$  builds up. This network is a clear example of a highly heterogeneous network, in that the degree distribution has unbounded fluctuations. As a reference of homogeneous networks, we considered the Watts-Strogatz (WS) network [18] in the case of complete random rewiring. In this case, one starts from a ring with  $N$  nodes, each of them connected symmetrically to  $2K$  neighbors. With probability  $p$ , each link connected to a clockwise neighbor is rewired to a randomly chosen node; otherwise it is preserved. After enough iterations, a random network with an exponential connectivity decay for large  $k$

and  $\langle k \rangle = 2K$  is generated. Henceforth, we will use  $m_0 = m = 3$  for the BA network and  $p = 1$  and  $K = 3$  for the WS model, giving  $\langle k \rangle = 6$  for both networks.

We have performed large-scale numerical simulations by applying repeatedly the rules stated above on BA and WS networks. Initially,  $\psi(0) = (N-1)/N$ ,  $\phi(0) = 1/N$ , and  $s(0) = 0$ , i.e., we start from a single spreader who is willing to spread the update through the network. At every time step, each of the  $\phi N$  spreaders contacts all its neighbors in a random sequence, unless during a contact it turns into a stifter. In this case it immediately stops contacting further nodes. This accounts for the larger transmission capabilities of high degree nodes that can reach a larger number of neighbors as specified by the heterogeneous network topology. The dynamical rules of the model are applied in parallel. The sizes of the networks used in the simulations carried out range from  $N = 10^3$  nodes to  $N = 10^5$  nodes and all numerical results have been obtained by averaging at least over 10 different networks and  $10^3$  iterations. The parameter  $\lambda$  may be varying as for the case of communication networks, where it is known that the rate of packet loss is not always zero. Nevertheless, without loss of generality,  $\lambda = 1$  since it just fixes the time scale by rescaling opportunely the rate  $1/\alpha$  in Eq. (2). On the other hand, we vary the rate at which spreaders decide not to communicate the update any more from  $\alpha = 1$  to  $\alpha = 10$  and monitor several quantities of interest.

In order to characterize the propagation process, we first focus on the *reliability*  $R$  of the rumor propagation defined as the final density  $s(\infty)$  of nodes that have gotten the update when the process dies out. For obvious practical purposes, any algorithm or process that emulates an efficient spreading of a given message or data packet will try to raise as much as possible this magnitude. In Table I, we report the reliability of the spreading process for the BA network and the random graph generated with the WS algorithm with  $p = 1$  for several values of the parameter  $\alpha$ . As noticed previously [13], in the WS network the number of stiflers at the end of the process is already high even for  $\alpha = 1$ , and the BA network appears less reliable. In general, it results that homogeneous networks allow a larger reliability  $R$  in epidemic updating processes. This is not straightforward and one may think that the existence of hubs in scale-free networks helps propagate the rumor. However, a closer look at the spreading dynamics tells us that the presence of hubs introduces conflicting effects in the dynamics. While hubs may in principle reach a larger number of nodes, spreader-spreader and spreader-stifler interactions are favored in the long run. Indeed, it is very unlikely that a hub in the spreader state contacts all its ignorant neighbors before turning into a stifter. Once a few hubs are turned into stiflers, many of the neighboring nodes

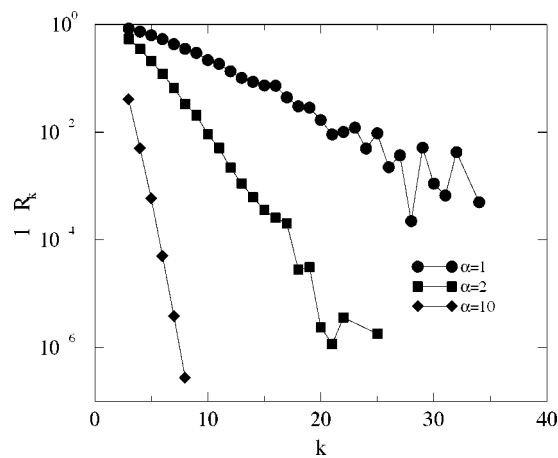


FIG. 1. Relative density of ignorants  $1 - R_k$  as a function of their connectivity  $k$  at the end of the spreading process in SF networks for different values of the parameter  $\alpha$ . The size of the network is always  $N = 10^4$  nodes.

could be isolated and never get the update. In this sense, homogeneous network allow for a more capillary diffusion of the update, since all nodes contribute equally to the message-passing. This is opposite to what happens in the usual epidemic spreading model in heterogeneous network models. These models lack an infected recovery rate induced by neighbors already infected and fully exploit the advantage of the hub's large degree [9].

The previous discussion, however, refers only to average properties. In SF networks, the connectivity distribution is highly heterogeneous and it is interesting to have more detailed insight into the reliability of the process for different connectivity classes. It may be particularly relevant that a higher  $R$  corresponds to the highly connected nodes, the hubs, which have a dominant role in the system. Figure 1 shows the behavior of the reliability  $R_k$  measured as a function of the nodes' connectivity  $k$ . This amounts to the relative density of nodes with connectivity  $k$  that have received the update and it is measured as  $R_k = \langle S_k / N_k \rangle$ , where  $S_k$  and  $N_k$  denote the total number of stiflers and the total number of nodes with degree  $k$ , respectively. The  $\langle \rangle$  represents the average over many realizations. The results confirm that during the spreading dynamics, it is very likely that highly connected nodes are reached by the update. In fact,  $1 - R_k$  decreases exponentially with  $k$  and very high levels of reliability are obtained well before the natural cutoff of the network ( $k_{\max} \sim 10^2$ ) even for moderate values of  $\alpha$ . This is an interesting feature signaling that heterogeneous topologies can be considered reliable as far as the "hubs" are concerned. We should note, however, that since the reliability is not a model-independent quantity, this result might depend upon the specific details of the rumor algorithm.

In general, one does not only want to have high reliability levels, but also the lowest cost in terms of network load [4,15]. This is generally achieved by imposing the minimum possible load to the network. Here, we define the load  $L$  imposed to the network as the number of contacts established per node, i.e., how many messages on average each node sends to its neighbors in order to propagate the update. By

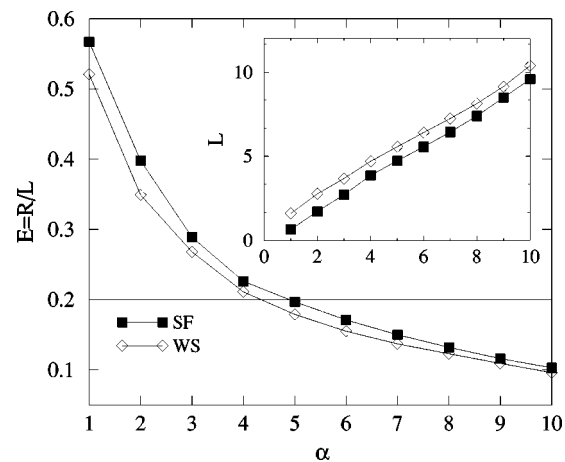


FIG. 2. Efficiency of the rumor-spreading process as a function of  $\alpha$  in networks of size  $N = 10^4$ . Results are compared with the efficiency of the basic broadcast algorithm that is represented by the thick line. The inset shows the growth of the load generated as a function of the parameter  $\alpha$ .

using this quantity, an obvious definition of the global efficiency  $E$  of the whole process is represented by the number of individuals who have gotten the update per unit of load,  $E = R/L$ . Its physical meaning is straightforward: the efficiency is equal to the fraction of "useful messages" (number of sites reached by the rumor) over total "load" imposed on the system. In Fig. 2, we report the behavior of the efficiency of the spreading process as a function of  $\alpha$ . In this case, the scale-free topology appears most efficient for data dissemination. Indeed, the relative difference between SF and WS networks in global efficiency is larger than 10% up to values of  $\alpha = 5$ . This can be appreciated also by looking at the inset in Fig. 2. For both topologies, the load on the network grows with  $\alpha$ , but the load imposed on SF networks is always smaller than on WS nets. Finally, it is interesting to compare the epidemic algorithm efficiency with those of the simplest broadcast strategy. This strategy essentially consists of a deterministic message-passing of each element to all its neighbors except the one from which the first update has been received. This way, a reliability  $R = 1$  is achieved since all nodes are surely contacted. In this case, the load is simply given by  $\langle k \rangle - 1$ . In the case of networks with  $\langle k \rangle = 6$  such as the one used in the present study, the efficiency of the broadcast strategy is therefore  $E = 0.2$ . It is interesting to note that in both the BA and the WS case, the epidemic algorithm achieves a better efficiency for a wide range of values of the parameter  $\alpha$ . This indicates that epidemic algorithms can provide attractive alternatives to broadcast solutions so far as the efficiency is concerned.

It is worth stressing that we are considering strategies in which the nodes do not have memory, i.e., they may try to resend a message to a node that has been contacted before. It is possible, however, to conceive different dynamics of the updating spreading strategies in which a tradeoff between the memory introduced in the process and the optimization of reliability and efficiency is opportunely chosen. Other options rely on a careful tuning of the local message-sending dynamics. For instance, we considered the case in which

each spreader contacts only one node at each time step, reducing the effects of the hubs. This simple change allows for higher levels of efficiency, however at the price of a much lower reliability. In general, it is thus possible to devise and tailor different processes that optimize one or more features of the update spreading on a given topology. We defer a more detailed study of this issue to future work as well as the inclusion of asynchronous effects resulting from communication delays.

In summary, we have studied the effect of the complex topological properties of many real networks in epidemic strategies for the communication of updates. The obtained results stimulate the seeking of heuristics and analytical methods to optimize epidemic algorithms, taking into account the specific topology of the underlying network. These studies may have a large impact in technological and communication networks where the use of rumorlike algorithms

might become a practice for data dissemination, reliable group communication, or replicated database maintenance. They might also provide a deeper understanding of social phenomena such as the spreading of new ideas in a population or the efficiency of marketing campaigns. Finally, we point out that the class of model studied here, including a time-delay mechanism, can be extended to the study of neuronal systems where a neuron's state can change as a result of its interaction with its neighboring nodes.

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